KEK-TH-990
UTHEP-499
hep-th/0410139

Open Semiclassical Strings and Long Defect Operators in AdS/dCFT Correspondence

Yoshiaki Susaki[*†a], Yastoshi Takayama[‡b] and Kentaroh Yoshida[*c]

[*] *Theory Division, Institute of Particle and Nuclear Studies,*
High Energy Accelerator Research Organization (KEK),
Tsukuba, Ibaraki 305-0801, Japan.

[†] *Institute of Physics, University of Tsukuba,*
Tsukuba, Ibaraki 305-8571, Japan.

[‡] *Department of Particle and Nuclear Physics,*
The Graduate University for Advanced Studies,
Tsukuba, Ibaraki 305-0801, Japan.

[a] susaki@post.kek.jp [b] takaya@post.kek.jp [c] kyoshida@post.kek.jp

Abstract

We consider defect composite operators in a defect superconformal field theory obtained by inserting an $\mathrm{AdS}_4\times\mathrm{S}^2$-brane in the $\mathrm{AdS}_5\times\mathrm{S}^5$ background. The one-loop dilatation operator for the scalar sector is represented by an integrable open spin chain. We give a description to construct coherent states for the open spin chain. Then, by evaluating the expectation value of the Hamiltonian with the coherent states in a long operator limit, a Landau-Lifshitz type of sigma model action is obtained. This action is also derived from the string action and hence we find a complete agreement in both SYM and string sides. We see that an $SO(3)_{\mathrm{H}}$ pulsating string solution is included in the action and its energy completely agrees with the result calculated in a different method. In addition, we argue that our procedure would be applicable to other AdS-brane cases.

Keywords: AdS/CFT, spin chain, integrability, semiclassical string, defect CFT

1 Introduction

One of the most important subjects in string theory is to test the AdS/CFT correspondence [1] beyond BPS sectors. In the analysis at almost BPS region, a great development was made by Berenstein-Maldacena-Nastase (BMN) [2]. They presented the AdS/CFT correspondence at string theoretical level by using the Penrose limit [3,4] and the exact solvability of pp-wave string theory [5]. The pp-wave string states and their energies correspond to the BMN operators and the full conformal dimensions (including the anomalous dimensions) in $\mathcal{N}$=4 super Yang-Mills (SYM), respectively.

The BMN analysis was generalized by considering the semiclassical quantization [6,7] around classical rotating string solutions, instead of pp-wave strings. The semiclassical string states and their energies correspond to certain single-trace local operators and their conformal dimensions in SYM in specific regimes.[1] The energy of the semiclassical string state can be expanded in positive power of new effective coupling $\tilde{\lambda} \equiv \lambda/J^2$ (λ is 't Hooft coupling) when the classical spinning string solutions have, at least, one of the S^5-spins J_1, J_2 and J_3. It is therefore possible to compare the energy with the conformal dimension perturbatively calculated in $\mathcal{N}$=4 SYM. Several types of classical solutions which give regular expansions of energies have been discovered by some works [6, 7, 11, 12] (For a review, see [13]). On the other hand, the anomalous dimensions of the single-trace composite operators can be computed by using the Bethe ansatz techniques. This fact was firstly shown by Minahan and Zarembo in a study of the $SO(6)$ scalar sector [14]. This kind of analysis is applicable to other sectors [15] such as $SU(2)$ [16–18], $SU(3)$ [19], $SL(2)$ [16,20–23][2], $SU(2|3)$ [24] and the full $PSU(2,2|4)$ sectors [22]. In particular, the $SU(2)$ sector is well studied. An approach to the analysis with the Inozemtsev long range spin chain at three-loop level [25] and, after that, the asymptotic all-loop Bethe ansatz[3] is proposed [26].

For the Bethe ansatz, in parallel to studies of quantum strings on the $AdS_5 \times S^5$, new types of

[1]The classical integrability of type IIB string on the $AdS_5 \times S^5$ background [8] may be deeply related to the correspondence between string and SYM sides [9]. The matching of the spectra and the equivalence of the integrable structures between the spin chain and string theories are confirmed up to and including the two-loop order on a specific example [10].

[2]The integrability related to the $SL(2)$ Heisenberg spin chain was previously found in non-supersymmetric QCD. For example, see [21].

[3]This is indeed asymptotic in a sense that it is unsuitable for describing the operators of small dimensions at sufficiently high orders of perturbation theory (e.g. certain interactions of wrapping type are missed).

Bethe ansatz for them were developed [27]. This important development should be a promising procedure to investigate the quantum aspects of the AdS-string.

In this paper we study the correspondence between semiclassical strings and long scalar operators in open string cases. By inserting spatial defects (called AdS-branes) in the bulk $AdS_5 \times S^5$ background [28], one may consider open strings in the context of the AdS/CFT correspondence where the CFT side has defects and so becomes a defect conformal field theory (dCFT). We focus on the case that an $AdS_4 \times S^2$-brane is inserted. The dCFT was particularly investigated by DeWolfe-Freedman-Ooguri [29] and it was shown to be superconformal [30]. The BMN operator correspondence for this system was also studied by Lee and Park [31]. Furthermore, the matrix of one-loop anomalous dimension for defect operators (which have the defect fields in the end-points, instead of the trace) in the scalar sector was calculated by DeWolfe and Mann [32][4]. The resulting dilatation operator is an integrable *open* spin chain Hamiltonian while a *closed* spin chain appears in the case of single trace operators. By using the Bethe ansatz techniques, the exact BMN operators [37] for the open strings were derived as in [14]. In the long operator limit, those are consistently reduced to the result of [31]. On the other hand, except the BMN operators, the string side is not uncovered yet.

Motivated by this fact, we will reveal the open semiclassical string corresponding to the open spin chain by using the method of [38, 39] with coherent state [40]. We give a description to construct coherent states for the defect operators and then the expectation value of the open spin chain Hamiltonian is taken with them. In a continuum limit (long operator limit), the Landau-Lifshitz (LL) type sigma model action is obtained as in [41–43]. Namely, the first time-derivative appears instead of the usual quadratic one. On the other hand, we derive the sigma model from the string action with a suitable gauge-fixing [42, 43]. In conclusion, we find a complete agreement of the sigma model actions as in [38, 39, 41–44]. We also see that the resulting sigma model action includes a pulsating solution and its energy is recovered by putting the solution ansatz into the action.

This paper is organized as follows: In section 2, the dCFT we consider is briefly introduced. Section 3 is devoted to a short review of the one-loop result in the scalar sector [32]. In section 4, we give a description to construct coherent states for the open spin chain Hamiltonian. Then we take the expectation value of the Hamiltonian, and the LL type sigma model is obtained in a

[4]For open strings in other setup, see [33, 34] and [35, 36].

continuum limit. We find that an $SO(3)_{\rm H}$ pulsating string solution is included in this action. In section 5, we rederive the sigma model obtained in section 4 from the string action. In section 6, we discuss coherent states for other AdS-brane cases. One may see that our consideration for an $\rm AdS_4{\times}S^2$ brane would be extended to other AdS-branes. Section 7 is devoted to a conclusion and discussions.

2 Setup of Defect Conformal Field Theory

From now on, we will consider a dCFT by inserting a supersymmetric $\rm AdS_4{\times}S^2$ brane into the $\rm AdS_5{\times}S^5$ background [29]. The $\rm AdS_4$ brane inside $\rm AdS_5$ could be naturally realized in string theory by considering a supersymmetric D5-brane intersecting a stack of N D3-branes as depicted in Fig. 1. The near-horizon limit of the N D3-branes produces an $\rm AdS_5{\times}S^5$ background where the D5-brane is realized as an $\rm AdS_4{\times}S^2$ submanifold. That is, in this system, closed strings propagating throughout the spacetime provide a holographic description of an $\mathcal{N}{=}4$ $\rm SYM_4$ on the boundary of $\rm AdS_5$. In addition, the fluctuations on the $\rm AdS_4$-brane should be dual to additional physics confined to the boundary of the $\rm AdS_4$, and the dual field theory contains new fields living on a $(1+2)$-dimensional defect, obtained from the low-energy limit of the 3-5 open strings interacting with the 3-3 strings of the original brane setup. The 5-5 strings are realized as open strings on the $\rm AdS_4{\times}S^2$ brane in the gravity side.

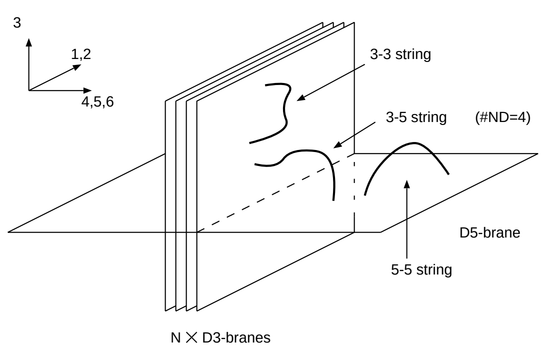

Fig. 1: An intersecting D5-brane with a stack of N D3-branes.

In the CFT side, $\mathcal{N}{=}4$ SYM has a three-dimensional defect due to the presence of the AdS-brane. One may see the usual $\mathcal{N}{=}4$ SYM in the long distance from the defect, while $\mathcal{N}{=}4$ SYM couples to the defect fields in the neighborhood of the defect. Then the vector multiplet of $\mathcal{N}{=}4$ in four dimensions decomposes into a vector multiplet and a hypermultiplet in three

dimensions.

Now let us introduce the field contents of $\mathcal{N}$=4 SYM and dCFT. The $\mathcal{N}$=4 vector multiplet is composed of a gauge field A_μ ($\mu = 0,1,2,3$), adjoint Majorana spinors λ^α ($\alpha = 1,2,3,4$) and six real scalars X^i ($i = 1,\ldots,6$). The λ^α and X^i are in the $\mathbf{4}$ and $\mathbf{6}$ of the $SO(6)$ R-symmetry, respectively. Our convention basically follows that of [32]. Then the defect introduces an additional 3D hypermultiplet propagating on the hypersurface $x^3 = 0$. This multiplet is composed of a complex scalar q^m and a complex 2-component fermion Ψ^a. These new fields break the total symmetry, and the breaking pattern is as follows:

$$\begin{aligned}
&\text{Conformal symmetry}: \quad SO(2,4) \to SO(2,3) \cong Sp(4)\,, \\
&\text{R-symmetry}: \quad SO(6)_\text{R} \to SO(3)_\text{H} \times SO(3)_\text{V} \cong O(4)\,, \\
&\text{Superconformal symmetry}: \quad PSU(2,2|4) \to OSp(4|4)\,.
\end{aligned}$$

The defect fields q^m and Ψ^a transform in the $(\mathbf{2},\mathbf{1})$ and in the $(\mathbf{1},\mathbf{2})$ of $SO(3)_\text{H} \times SO(3)_\text{V}$, respectively. The $\mathcal{N}$=4 vector multiplet is also decomposed into

$$\begin{aligned}
&\text{3D vector multiplet}: \quad \{A_k, P_+\lambda^\alpha, X_\text{V}^A, D_3 X_\text{H}^I\}\,, \\
&\text{3D hypermultiplet}: \quad \{A_3, P_-\lambda^\alpha, X_\text{H}^I, D_3 X_\text{V}^A\}\,,
\end{aligned}$$

where $k = 0,1,2$ and $i = 1,\ldots,6$ of $SO(6)$ is decomposed into $A = 1,2,3$ of $\mathbf{3}$ in the $SO(3)_\text{V}$ and $I = 4,5,6$ of $\mathbf{3}$ in the $SO(3)_\text{H}$. That is, X_H^I and X_V^A are in the $(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{3})$ of the $SO(3)_\text{H} \times SO(3)_\text{V}$, respectively.

These defects couple directly only to the bulk vector multiplet, and their dynamics is described by the following action[5]

$$\begin{aligned}
S_3 &= S_\text{kin} + S_\text{yuk} + S_\text{pot}\,, \qquad (2.1)\\
S_\text{kin} &= \frac{1}{g^2}\int d^3x \Big[(D^k q^m)^\dagger D_k q^m - i\bar{\Psi}^a \rho^k D_k \Psi^a\Big]\,, \\
S_\text{yuk} &= \frac{1}{g^2}\int d^3x \Big[i\bar{\Psi}^a P_+ \lambda_{am} q^m - i\bar{q}^m \bar{\lambda}_{ma} P_+ \Psi^a + \bar{\Psi}^a \sigma^A_{ab} X^A_\text{V} \Psi^a\Big]\,, \\
S_\text{pot} &= \frac{1}{g^2}\int d^3x \Big[\bar{q}^m X^A_\text{V} X^A_\text{V} q^m + i\epsilon_{IJK}\bar{q}^m \sigma^I_{mn} X^J_\text{H} X^K_\text{H} q^n\Big] \\
&\quad + \frac{1}{g^2}\int d^3x \left[\bar{q}^m \sigma^I_{mn}(D_3 X^I_\text{H}) q^n + \frac{1}{4}\delta(0)\text{Tr}(\bar{q}^m \sigma^I_{mn} q^n)^2\right]\,,
\end{aligned}$$

[5]We will consider the case that a single D5-brane intersecting with a stack of N D3-branes in this paper. For $M \ll N$, we may consider the case that M multiple D5-branes intersect with N D3-branes, and flavor indices appear on q and Ψ in an obvious way, while the $M \sim N$ case is more involved and difficult because the backreaction of the D5-branes should be considered.

where the covariant derivative is defined as $D_k * \equiv \partial_k * - iA_k *$. The total action of the theory is given by the action of $\mathcal{N}$=4 SYM$_4$ and the above action for the defect fields. Notably, the total theory has only one parameter, coupling $g_{\rm YM}$, and so it is exactly marginal [30].

3 The One-Loop Anomalous Dimensions of Defect Operators

Here we shall briefly review the one-loop result obtained by DeWolfe and Mann [32]. We are interested in open strings on the AdS$_4\times$S^2-brane. Their states are described by local, gauge-invariant defect operators composed of Lorentz scalar fields:

$$\mathcal{O} = \psi_{m,j_1,\ldots,j_L,n}\, \bar{q}_m X^{j_1} \cdots X^{j_L} q^n \,. \tag{3.1}$$

Here two defect scalar fields appear at both ends of a sequence of scalar fields, instead of the trace operation in closed string cases. By evaluating the correlation function,

$$\langle \bar{q}_{n'}(z_\beta) X^{i_L}(z_L) \cdots X^{i_1}(z_1) q^{m'}(z_\alpha) \mathcal{O}(0) \rangle \,, \tag{3.2}$$

one can compute the matrix of anomalous dimension for (3.1). In the one-loop level analysis, the bulk interactions lead to the same result in closed string cases, but the defect interactions give additional terms to the matrix of anomalous dimension.

The matrix of anomalous dimension at one-loop level is computed by evaluating the one-loop planar diagrams. It is represented by the Hamiltonian of an integrable open spin chain and it is composed of the bulk part and the defect part as follows:

$$D_{SO(6)} = \Gamma^{\rm bulk}_{\mathcal{O}} + \Gamma^{\rm defect}_{\mathcal{O}} = \frac{\lambda}{16\pi^2} \sum_{l=1}^{L-1} H_{l,l+1} + \frac{\lambda}{16\pi^2} \Big[(2I_{\bar{\alpha}1} + 2\bar{S}_{\bar{\alpha}1}) + (2I_{L\beta} + 2S_{L\beta}) \Big] \,, \tag{3.3}$$

$$H_{l,l+1} = K_{l,l+1} + 2I_{l,l+1} - 2P_{l,l+1} \,. \tag{3.4}$$

The bulk part (the first term in (3.3)) is the same as the result of Minahan and Zarembo [14], and it is written by the trace I, permutation P and trace K operators:

$$I^{i_l i_{l+1}}_{j_l j_{l+1}} = \delta^{i_l}_{j_l} \delta^{i_{l+1}}_{j_{l+1}} \,, \quad P^{i_l i_{l+1}}_{j_l j_{l+1}} = \delta^{i_l}_{j_{l+1}} \delta^{i_{l+1}}_{j_l} \,, \quad K^{i_l i_{l+1}}_{j_l j_{l+1}} = \delta^{i_l i_{l+1}} \delta_{j_l j_{l+1}} \,. \tag{3.5}$$

The defect part (the second term in (3.3)) is written in terms of

$$\begin{aligned} &I^{\bar{m}I}_{\bar{n}J} = \delta^{\bar{m}}_{\bar{n}} \delta^I_J \,, \quad I^{Im}_{Jn} = \delta^I_J \delta^m_n \,, \quad I^{\bar{m}A}_{\bar{n}B} = I^{\bar{m}I}_{\bar{n}B} = I^{\bar{m}A}_{\bar{n}J} = 0 \,, \quad I^{Bn}_{Am} = I^{Jn}_{Am} = I^{Bn}_{Im} = 0 \,, \\ &S^{Im}_{Jn} = -i\epsilon_{IJK} \sigma^K_{mn} \,, \quad S^{Am}_{Bn} = \delta^m_n \delta^A_B \,, \quad S^{Im}_{Bn} = S^{Am}_{Jn} = 0 \,, \\ &\bar{S}^{\bar{m}I}_{\bar{n}J} = i\epsilon_{IJK} \sigma^K_{\bar{m}\bar{n}} \,, \quad \bar{S}^{\bar{m}A}_{\bar{n}B} = \delta^{\bar{m}}_{\bar{n}} \delta^A_B \,, \quad \bar{S}^{\bar{m}I}_{\bar{n}B} = \bar{S}^{\bar{m}A}_{\bar{n}J} = 0 \,. \end{aligned} \tag{3.6}$$

Here we should note that the bulk part has no periodicity. In the present case, the integrable boundary arising from the defect contribution ensures the integrability of the spin chain, instead of periodicity.

It may be helpful for a check of calculations to use the chiral primary operators [29],

$$\bar{q}^m \sigma^{(I_1}_{mn} X^{I_2}_{\rm H} X^{I_3}_{\rm H} \cdots X^{I_{L+1})}_{\rm H} q^n \,, \tag{3.7}$$

where parentheses denote total symmetrization and traceless with respect to the index of $SO(3)_{\rm H}$. One can indeed verify that each $H_{l,l+1}$, as well as $2I_{\bar{\alpha}1} + 2\bar{S}_{\bar{\alpha}1}$ and $2I_{L\beta} + 2S_{L\beta}$, separately gives zero by acting them to (3.7).

Finally, by introducing the $SO(6)$ generators $M^{ij}_{ab} \equiv \delta^i_a \delta^j_b - \delta^j_a \delta^i_b$, a piece of the spin chain Hamiltonian, $H_{l,l+1}$ is rewritten as

$$H_{l,l+1} = M^{ij}_l M^{ij}_{l+1} - \frac{1}{16}(M^{ij}_l M^{ij}_{l+1})^2 + \frac{9}{4}\,. \tag{3.8}$$

In this form the spin-spin interaction is manifest. The expression (3.8) is useful to evaluate the expectation value of the spin chain Hamiltonian with coherent states.

4 Coherent States and Sigma Model for Defect Operators

4.1 Construction of Coherent States for Open Spin Chain

We will consider how to construct the coherent states for the open spin chain. The open spin chain has boundaries which break the $SO(6)$ symmetry to $SO(3)_{\rm H} \times SO(3)_{\rm V}$. This fact reflects that a D5-brane, whose shape is an $\mathrm{AdS}_4 \times \mathrm{S}^2$ geometry, is inserted into the $\mathrm{AdS}_5 \times \mathrm{S}^5$. We may consider various open semiclassical strings attaching to the S^2 part, but we concentrate on open strings sticking to the S^2 described in the three-dimensional space spanned by $X^I_{\rm H}$ $(I = 1, 2, 3)$ with the condition $\sum_{I=1}^3 (X^I_{\rm H})^2 = 1$. It would be difficult to obtain a regular semiclassical string energy (i.e., a regular BMN limit) in the other cases. In fact, we have not succeeded so far.

Hence let us consider the following ansatz for the whole coherent state:

$$|m\rangle = |\bar{q}\rangle \otimes \prod_{l=1}^{L} |m_l\rangle \otimes |q\rangle \,, \tag{4.1}$$

where $|m_l\rangle$ are $SO(3)_{\rm H}$ coherent states, and $|\bar{q}\rangle$, $|q\rangle$ are $SU(2)_{\rm H}$ ones. We need to construct and adjust the these coherent states so that the well-defined continuum limit can be taken.

For the $SO(3)_{\rm H}$ sector, the coherent states are described by using a coset $SO(3)_{\rm H}/SO(2)$. Then, according to the choices of the $SO(6)$ coherent states, there are two possibilities:

1. the vacuum $|0\rangle = (0,1,i)$ and $H = SO(2)$ where the vacuum is invariant up to a phase factor. The Cartan generator is M_{56} and the coherent state is generated by M_{45} and M_{64} .

2. the vacuum $|0\rangle = (0,0,1)$ and $H = SO(2)$. The Cartan generator is M_{45} and the coherent state is generated by M_{56} and M_{64} .

Here we have extracted the vacuum and the generators for $SO(3)_{\mathrm{H}}/SO(2)$ from the full $SO(6)$ case. As we will see below, the "local" BPS property (case 1) and the large "extensive" one-loop shift of the dimension (case 2) are also inherited even after the $SO(6)$ is spontaneously broken. We will concentrate on the case 1. In the end of this section we will comment on the case 2. We will explain the construction of $SO(3)_{\mathrm{H}}/SO(2)$ and $SU(2)_{\mathrm{H}}/U(1)$ coherent states below.

Construction of $SO(3)_{\mathrm{H}}/SO(2)$ coherent states

Let us introduce M^{IJ} as $SO(3)_{\mathrm{H}}$ generators in the fundamental representation $(M^{IJ})_{ab} = \delta^I_a\delta^J_b - \delta^I_b\delta^J_a$ $(a,b = 1,2,3,\ I,J = 1,2,3)$[6]. Choosing a vector $(1,i,0)$ as a vacuum $|0\rangle$ (i.e., the highest weight state for $M^+ = M^{23} + iM^{31}$) , the $SO(3)_{\mathrm{H}}/SO(2)$ coherent state is given by

$$|m\rangle = \boldsymbol{m} \equiv \frac{1}{\sqrt{2}}\exp[aM^{23} + bM^{31}]\begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix} = \frac{\mathrm{e}^{-i\phi}}{\sqrt{2}}\begin{bmatrix} \cos\theta\cos\phi + i\sin\phi \\ -\cos\theta\sin\phi + i\cos\phi \\ \sin\theta \end{bmatrix}, \tag{4.2}$$

where $\theta = \sqrt{a^2+b^2}$, $a = \theta\sin\phi$ and $b = \theta\cos\phi$. One can explicitly check that $\langle m|m\rangle = |\boldsymbol{m}|^2 = 1$, $\quad \boldsymbol{m}^2 = (\boldsymbol{m}^*)^2 = 0$. By putting this coherent state at each l-th site as $|m_l\rangle$,

$$|m_l\rangle = \boldsymbol{m}_l = \frac{\mathrm{e}^{-i\phi_l}}{\sqrt{2}}\begin{bmatrix} \cos\theta_l\cos\phi_l + i\sin\phi_l \\ -\cos\theta_l\sin\phi_l + i\cos\phi_l \\ \sin\theta_l \end{bmatrix} \qquad (l = 1,\ldots,L)\,, \tag{4.3}$$

the $SO(3)_{\mathrm{H}}$ coherent states for the bulk part of the spin chain are constructed.

Construction of $SU(2)_{\mathrm{H}}/U(1)$ coherent states

We have to determine two more coherent states $|\bar{q}\rangle$ and $|q\rangle$. The $SU(2)_{\mathrm{H}}/U(1)$ coherent states can be written, up to a $U(1)$ factor, as

$$\left|\frac{SU(2)_{\mathrm{H}}}{U(1)}\right\rangle = \mathrm{e}^{i\alpha\sigma_1 + i\beta\sigma_2}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\vartheta/2) \\ \mathrm{e}^{-i\varphi}\sin(\vartheta/2) \end{bmatrix} \simeq \begin{bmatrix} \mathrm{e}^{i\varphi/2}\cos(\vartheta/2) \\ \mathrm{e}^{-i\varphi/2}\sin(\vartheta/2) \end{bmatrix}, \tag{4.4}$$

[6]We have redefined the expressions of the $SO(3)_{\mathrm{H}}$ generators as M_{45}, M_{56}, $M_{64} \to M_{31}$, M_{12}, M_{23}.

where $\vartheta = 2\sqrt{\alpha^2+\beta^2}\,,\ \alpha = \frac{\vartheta}{2}\sin\varphi\,,\ \beta = \frac{\vartheta}{2}\cos\varphi\,.$ The vacuum $(1,0)$ is the highest weight state for $\sigma_+ = \sigma_1 + i\sigma_2\,.$ We take $|\bar{q}\rangle$ and $|q\rangle$ as

$$|\bar{q}\rangle = \begin{bmatrix} \mathrm{e}^{-i\varphi_{\bar{q}}/2}\cos(\vartheta_{\bar{q}}/2) \\ \mathrm{e}^{i\varphi_{\bar{q}}/2}\sin(\vartheta_{\bar{q}}/2) \end{bmatrix}\,, \qquad |q\rangle = \begin{bmatrix} \mathrm{e}^{i\varphi_q/2}\cos(\vartheta_q/2) \\ \mathrm{e}^{-i\varphi_q/2}\sin(\vartheta_q/2) \end{bmatrix}\,. \tag{4.5}$$

These coherent states satisfy

$$\langle\bar{q}|\boldsymbol{\sigma}|\bar{q}\rangle = \langle q|\boldsymbol{\sigma}^T|q\rangle = \big(\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta\big) \equiv \boldsymbol{n}\,. \tag{4.6}$$

We next discuss the expectation values of $\Gamma_{\mathcal{O}}^{\text{defect}}$. The boundary parts are described as

$$|\mathcal{L}\rangle = |m_1\rangle \otimes |\bar{q}\rangle\,, \qquad |\mathcal{R}\rangle = |m_L\rangle \otimes |q\rangle\,. \tag{4.7}$$

For the left end-point, the expectation value is[7]

$$\begin{aligned} \langle\mathcal{L}|\Gamma_{\mathcal{O}}^{\text{defect}-\text{L}}|\mathcal{L}\rangle &\simeq \langle\mathcal{L}|I_{\bar{q}1} + S_{\bar{q}1}|\mathcal{L}\rangle = 1 + i\boldsymbol{n}_{\bar{q}}\cdot(\boldsymbol{m}_1^*\times\boldsymbol{m}_1) \\ &= 1 - (\sin\vartheta_{\bar{q}}\cos\varphi_{\bar{q}}, \sin\vartheta_{\bar{q}}\sin\varphi_{\bar{q}}, \cos\vartheta_{\bar{q}}) \begin{bmatrix} \sin\theta_1\cos(\pi-\phi_1) \\ \sin\theta_1\sin(\pi-\phi_1) \\ \cos\theta_1 \end{bmatrix}\,. \end{aligned} \tag{4.8}$$

In a similar way, for the right end-point, we obtain

$$\begin{aligned} \langle\mathcal{R}|\Gamma_{\mathcal{O}}^{\text{defect}-\text{R}}|\mathcal{R}\rangle &\simeq \langle\mathcal{R}|I_{Lq} + S_{Lq}|\mathcal{R}\rangle = 1 - i\boldsymbol{n}_q\cdot(\boldsymbol{m}_L^*\times\boldsymbol{m}_L) \\ &= 1 + (\sin\vartheta_q\cos\varphi_q, \sin\vartheta_q\sin\varphi_q, \cos\vartheta_q) \begin{bmatrix} \sin\theta_L\cos(\pi-\phi_L) \\ \sin\theta_L\sin(\pi-\phi_L) \\ \cos\theta_L \end{bmatrix}\,. \end{aligned} \tag{4.9}$$

Now let us consider the following conditions:

$$\left\{ \begin{array}{c} \vartheta_{\bar{q}} = \theta_1 + \dfrac{1}{L}\delta\theta_1 + \dfrac{1}{L^2}\Delta\theta_1 \\ \varphi_{\bar{q}} = \pi - \phi_1 - \dfrac{1}{L}\delta\phi_1 - \dfrac{1}{L^2}\Delta\phi_L \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{c} \vartheta_q = \pi + \theta_L + \dfrac{1}{L}\delta\theta_L + \dfrac{1}{L^2}\Delta\theta_L \\ \varphi_q = \pi - \phi_L - \dfrac{1}{L}\delta\phi_L - \dfrac{1}{L^2}\Delta\phi_L \end{array} \right. . \tag{4.10}$$

In general $\langle\mathcal{L}|\Gamma^{\text{defect}-\text{L}}|\mathcal{L}\rangle$ is expanded in terms of $1/L$ as

$$\langle\mathcal{L}|\Gamma^{\text{defect}-\text{L}}|\mathcal{L}\rangle = \lambda\left\{a_0 + a_1\frac{1}{L} + a_2\frac{1}{L^2}\right\} + \mathcal{O}\left(\frac{1}{L^3}\right)\,, \tag{4.11}$$

[7] For a moment, we omit the factor $\frac{\lambda}{8\pi^2}$ for convenience since it is irrelevant here.

but under the conditions (4.10) the constants a_0 and a_1 vanish. In other words, the leading parts of (4.10) are determined so that we can take a regular BMN limit. Then the terms with higher order than $1/L^3$ also vanish in the $L\to\infty$ limit. The second-order term gives only a non-vanishing contribution

$$\langle\mathcal{L}|\Gamma^{\rm defect-L}|\mathcal{L}\rangle = \frac{\lambda}{8\pi^2}\times\frac{1}{2L^2}\Big\{(\delta\theta_1)^2+\sin^2\theta_1(\delta\phi_1)^2\Big\} \tag{4.12}$$

$$= \frac{1}{4\pi^2}\frac{\tilde{\lambda}}{2}\Big\{|\delta\boldsymbol{m}_1|^2-|\boldsymbol{m}_1^*\cdot\delta\boldsymbol{m}_1|^2\Big\}. \tag{4.13}$$

This will be absorbed into the action obtained from the bulk contribution. The result that the non-diagonal integrable boundary terms give the same action as the bulk part would be rather non-trivial, though it is physically quite natural. The same argument can be applied for $\langle\mathcal{R}|\Gamma^{\rm defect-R}|\mathcal{R}\rangle$.

The Dirichlet and Neumann boundary conditions for open strings may be encoded into δm_1 in (4.13) . Those cannot be however determined before taking the continuum limit because the σ-dependence of $\boldsymbol{m}$ should be explicitly specified. It is necessary to take a variation of the continuum action in order to determine the boundary conditions.

Finally we shall give a simple geometrical interpretation of the conditions $a_0=a_1=0$. We focus on the boundary on the left hand side since the analysis on the other side is similar. The key point is that $n_0^K\equiv\langle\bar{q}|\sigma^K|\bar{q}\rangle$ should be interpreted as the coordinates for the end-point of the open string on S^2 [35]. Since $(L^K)_{I_1J_1}\equiv i\epsilon_{I_1KJ_1}$ are the $SO(3)_{\rm H}$ generators, $n_1^K\equiv\langle m_1|L^K|m_1\rangle$ should denote the nearest point from the end-point. Then we find that $a_0=0$ leads to $1-\boldsymbol{n}_0\cdot\boldsymbol{n}_1=0$. In the continuum limit, $\boldsymbol{n}_0$ and $\boldsymbol{n}_1$ are very close and hence we can expand as $\boldsymbol{n}_1\simeq\boldsymbol{n}_0+d\boldsymbol{n}_0$. Then the condition $a_1=0$ leads to $\boldsymbol{n}_0\cdot d\boldsymbol{n}_0=0$. Thus the conditions $a_0=a_1=0$ imply that the endpoints of the open string should lie on the S^2-brane.

4.2 Derivation of Landau-Lifshitz type Sigma Model from the dCFT

Now let us consider the expectation value of the open spin chain Hamiltonian in terms of coherent states (4.3), (4.5).

The expectation value of $D_{SO(6)}$ in the whole coherent states $|m\rangle$ consists of three parts as

$$\langle m|D_{SO(6)}|m\rangle = \langle m|\Gamma^{\rm bulk}_{\mathcal{O}}|m\rangle+\langle\mathcal{L}|\Gamma^{\rm defect-L}_{\mathcal{O}}|\mathcal{L}\rangle+\langle\mathcal{R}|\Gamma^{\rm defect-R}_{\mathcal{O}}|\mathcal{R}\rangle\,. \tag{4.14}$$

Here the defect contributions have been already evaluated and the next task is to evaluate the bulk contribution. Following the procedure in [41], we shall introduce an antisymmetric 3×3

matrix m^{IJ} defined as

$$m^{IJ} \equiv \langle m|M^{IJ}|m\rangle = m^{a*}M^{IJ}_{ab}m^b = m^{I*}m^J - m^I m^{J*} \quad (I,J=1,2,3)\,. \tag{4.15}$$

The expectation value of $H_{l,l+1}$ is evaluated as

$$\begin{aligned}\langle m|H_{l,l+1}|m\rangle &= \frac{15}{16}\sum_{I,J=1}^{3} m_l^{IJ}m_{l+1}^{IJ} - \frac{1}{16}\sum_{I,J,K,L=1}^{3} m_l^{IJ}m_l^{JK}m_{l+1}^{KL}m_{l+1}^{LI} \\ &= \frac{15}{32}\mathrm{Tr}(m_l - m_{l+1})^2 + \frac{1}{32}\mathrm{Tr}[(m_l^2 - m_{l+1}^2)^2]\,. \end{aligned} \tag{4.16}$$

Then we consider a continuum limit $L\to\infty$ with $\tilde{\lambda}\equiv\frac{\lambda}{L^2}$ fixed. We expand in powers of π/L as

$$m^{IJ}(\sigma_{l+1}) = m^{IJ}(\sigma_l) + \frac{\pi}{L}\partial_\sigma m^{IJ} + \cdots\,, \tag{4.17}$$

and keep the leading terms. The continuum limit is taken as

$$\begin{aligned}\langle m|D_{SO(6)}|m\rangle &\to \frac{\lambda}{16\pi^2}L\int_0^\pi\frac{d\sigma}{\pi}\left(\frac{\pi}{L}\right)^2\mathrm{Tr}\left[\frac{15}{32}(\partial_\sigma m)^2 + \frac{1}{16}m^2(\partial_\sigma m)^2 + \frac{1}{16}(m\partial_\sigma m)^2\right] \\ &\simeq L\int_0^\pi\frac{d\sigma}{\pi}\frac{\tilde{\lambda}}{32}\mathrm{Tr}\left[(\partial_\sigma m)^2 + \frac{1}{16}(m\partial_\sigma m)^2\right]\,, \end{aligned} \tag{4.18}$$

where we have used $\mathrm{Tr}(\partial_\sigma m)^2 = 2\mathrm{Tr}\left[m^2(\partial_\sigma m)^2\right] + \mathrm{Tr}\left(m\partial_\sigma m\right)^2$. By substituting (4.15) into (4.18),

$$\begin{aligned}\langle m|D_{SO(6)}|m\rangle &\simeq \frac{1}{4}L\int_0^\pi\frac{d\sigma}{\pi}\frac{\tilde{\lambda}}{2}\Big\{|\partial_\sigma\boldsymbol{m}|^2 - |\boldsymbol{m}^*\cdot\partial_\sigma\boldsymbol{m}|^2\Big\} \\ &= -L\int_0^\pi\frac{d\sigma}{\pi}\Big\{-\frac{1}{2}\cdot\frac{\tilde{\lambda}}{4}|D_\sigma\boldsymbol{m}|^2\Big\} \equiv -L\int_0^\pi\frac{d\sigma}{\pi}\langle m|\mathcal{H}|m\rangle\,. \end{aligned} \tag{4.19}$$

The phase-space Lagrangian $\mathcal{L}$ is defined as

$$\mathcal{L} \equiv -\langle m|i\frac{d}{dt}|m\rangle + \langle m|\mathcal{H}|m\rangle\,, \tag{4.20}$$

and the resulting sigma model action is given by

$$I = L\int dt\int_0^\pi\frac{d\sigma}{\pi}\,\mathcal{L}\,, \qquad \mathcal{L} = -i\boldsymbol{m}^*\cdot\partial_t\boldsymbol{m} - \frac{1}{2}\cdot\frac{\tilde{\lambda}}{4}|D_\sigma\boldsymbol{m}|^2\,. \tag{4.21}$$

In the next section we will reproduce the same result as (4.21) from the string side.

Comment on the $SO(6)/SO(5)$ case

We may consider the case 2 related to $H = SO(5)$ in the unbroken $SO(6)$ case. When we follow the above analysis in this case, we cannot completely delete the contribution of the additional anomalous dimension because the ϵ_{IJK} terms vanish due to the reality of the $SO(3)_{\rm H}$ coherent state (i.e. $\langle m|_I \epsilon_{IJK}|m\rangle_K = 0$). Thus the Kronecker delta terms still remain even after taking the expectation value of the spin chain Hamiltonian, and lead to the term proportional to λ. This contribution is absorbed into the large "extensive" one-loop shift that comes from the bulk contribution, but the problem of the consistent continuum limit still remains. After all, the problem of the defect interactions reduces to that of the continuum limit.

5 Landau-Lifshitz Type Sigma Model from the String Action

5.1 Derivation of Landau-Lifshitz Type Sigma Model

We shall derive a Landau-Lifshitz type sigma model action as an effective action of the open string moving almost at the speed of light, following [42]. In our case, open strings can move only on the S^2 spanned by $X^I_{\rm H}$ $(I = 1, 2, 3)$ with $\sum_{I=1}^3 (X^I_{\rm H})^2 = 1$. Hence we start from the bosonic part of the phase-space Lagrangian for an open string on $R\times\mathrm{S}^2$,

$$\mathcal{L} = -\frac{1}{2}\kappa^2 + P^I_{\rm H}\dot{X}^I_{\rm H} - \frac{1}{2}P^I_{\rm H}P^I_{\rm H} - \frac{1}{2}X^I_{\rm H}{}'X^I_{\rm H}{}' - \frac{1}{2}\Lambda(X^I_{\rm H}X^I_{\rm H} - 1)\,, \tag{5.1}$$

where the symbols "$\cdot$" and "$\prime$" mean the derivatives with respect to the world-sheet coordinates τ and σ, respectively. Here we have chosen the conformal gauge and the additional gauge-fixing condition $t = \kappa\tau$.

Next we consider an effective action where the slowly-changing coordinates only survive, and so we first isolate a fast coordinate. In this process, it is inevitable to use the phase-space description. Then we take a limit in which the velocity of the fast coordinate is put to infinity. Finally we get the following Landau-Lifshitz type effective action

$$I = L\int dt \int_0^\pi \frac{d\sigma}{\pi}\,\mathcal{L}\,, \quad \mathcal{L} = -iV^*\dot{V} - \frac{1}{2}|D_\sigma V|^2\,, \tag{5.2}$$

where $L = \sqrt{\lambda}\,l$ $(l \simeq \kappa)$ and the symbol "$\cdot$" implies the derivative with respect to the time t, instead of τ. The time coordinate is rescaled as $t \to \kappa^2 t$ and the Lagrange multiplier terms are omitted. In addition, when we rescale the time t as $t \to \frac{1}{4}\tilde{\lambda}\,t$,[8] the Wess-Zumino term is kept

[8] The factor $1/4$ arises since $\tilde{\lambda} \equiv \lambda/L^2$ and the L for closed strings is replaced by $2L$ for open strings.

invariant but the remaining part is shifted as

$$I = L\int dt \int_0^\pi \frac{d\sigma}{\pi}\,\mathcal{L}\,, \quad \mathcal{L} = -iV^*\dot{V} - \frac{1}{2}\cdot\frac{\tilde{\lambda}}{4}|D_\sigma V|^2\,. \tag{5.3}$$

We thus find a complete agreement of the sigma model actions (4.21) in SYM and (5.3) in the string side, under the identification of $\boldsymbol{m}$ with V.

In the closed string case, the Landau-Lifshitz type sigma model can also be derived from the approach of Mikhailov [48]. It would be an interesting practice to rederive the action in our open string case, by following [48].

5.2 $SO(3)_{\rm H}$ *Pulsating String Solution*

We can find a pulsating solution under a solution ansatz. Let us parametrize the complex vector V^I as

$$V^I = \frac{a^I - ib^I}{\sqrt{2}}\,, \quad V^I V^{I*} = 1\,, \quad (V^I)^2 = 0\,. \tag{5.4}$$

By setting the real vectors a and b as

$$a = (\cos\theta\cos\phi, -\cos\theta\sin\phi, \sin\theta)\,, \quad b = (-\sin\phi, -\cos\phi, 0)\,, \tag{5.5}$$

the Lagrangian becomes

$$\mathcal{L} = \cos\theta\,\dot{\phi} - \frac{\tilde{\lambda}}{16}\left(\theta'^2 + \sin^2\theta\,\phi'^2\right)\,, \tag{5.6}$$

in terms of two coordinates θ and ϕ of S^2. The pulsating solution is now described by taking a special solution $\theta = \pi/2$ and $\phi = m\sigma$ where m is an integer[9]. The energy of the pulsating string solution is obtained as

$$E = \frac{1}{16}\tilde{\lambda}L\sin^2\theta\,\phi'^2 = \frac{\lambda}{16L}m^2\,. \tag{5.7}$$

It completely agrees with the result of [45] in the open string case. Here we have utilized the doubling trick formula that is discussed by Stefanski [35],

$$E_{\rm open}(L) = \frac{1}{2}E_{\rm closed}(2L)\,. \tag{5.8}$$

Then the energy for the pulsating closed string solution is

$$E_{\rm closed} = \frac{\lambda}{4L}m^2\,, \tag{5.9}$$

[9] For more general pulsating string solutions, see [46, 47].

and hence the doubling trick formula (5.8) is surely satisfied.

We should remark that the pulsating closed string solution is also a solution in our open string case by imposing Neumann and Dirichlet conditions at the end-points:

$$\mathrm{N}: \quad \partial_\sigma V(t,\sigma=0,\pi)=0\,, \qquad \mathrm{D}: \quad V(t,\sigma=0,\pi)=0\,. \tag{5.10}$$

In the parametrization of (5.5) and the solution ansatz: $\theta=\pi/2$ and $\phi=m\sigma$, V^2 and V^3 satisfy Neumann conditions, and V^1 does a Dirichlet one.

5.3 Construction of Coherent Defect Operators

Finally we shall comment on the forms of defect operators corresponding to open string solutions. The defect operators naturally associated to open semiclassical string solutions should be "locally BPS" coherent defect operators. Slightly generalizing the factorization ansatz in the closed string case [42], we suppose that the coefficient of (3.1) is decomposed as

$$\psi_{mI_1I_2\cdots I_Ln}=\eta'_m m_1^{I_1}m_2^{I_2}\cdots m_L^{I_L}\eta_n\,. \tag{5.11}$$

Here m_l's are the coherent states (4.3) and correspond to open string solutions under the identification of $\boldsymbol{m}$ with V in the continuum limit. By assuming that η'_m and η_n are given as $(\eta'_m)=|\bar{q}\rangle$ and $(\eta_n)=|q\rangle$, the corresponding operators are

$$\mathcal{O}=[\eta'_m\bar{q}_m]\left[\prod_{l=1}^{L}m_l^{I_l}X_{\mathrm{H}}^{I_l}\right][\eta_n q_n]\,. \tag{5.12}$$

In particular, when we set $\theta_l=\phi_l=0$ for all l ,

$$(\eta'_m)=\begin{bmatrix}1\\0\end{bmatrix}\,,\quad (\eta_n)=\begin{bmatrix}0\\1\end{bmatrix}\,,\qquad (m_l^{I_l})=\begin{bmatrix}1\\i\\0\end{bmatrix}\quad (l=1,\cdots,L)\,, \tag{5.13}$$

we can consistently reproduce the open string BPS vacuum [31]

$$\mathcal{O}=\bar{q}_1Z^Lq_2 \qquad (Z\equiv X_{\mathrm{H}}^1+iX_{\mathrm{H}}^2)\,. \tag{5.14}$$

In addition we may construct the operator corresponding to the pulsating open string solution discussed above by putting the following data into (5.12) ,

$$(\eta'_m)=\frac{i}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}\,,\quad (\eta_n)=\frac{i^{m+1}}{\sqrt{2}}\begin{bmatrix}1\\(-1)^m\end{bmatrix}\,,\quad (m_l^{I_l})=\frac{e^{-im\sigma_l}}{\sqrt{2}}\begin{bmatrix}i\sin(m\sigma_l)\\i\cos(m\sigma_l)\\1\end{bmatrix}\,. \tag{5.15}$$

Here a few examples have been discussed, but we need more efforts to confirm the ansatz (5.12) .

6 Other AdS-brane Cases

We have discussed the case that an $AdS_4\times S^2$-brane is inserted in the $AdS_5\times S^5$ background. By considering other AdS-branes, one may obtain other defect conformal field theories (For the classification of possible AdS-branes, see the work of Skenderis and Taylor [49] or an approach from κ-symmetry [50]). The possible 1/2 supersymmetric AdS-branes are summarized in Tab. 1. For all cases, the holographic duals are described by dCFTs. The feature of the defect fields is different for the number of Neumann-Dirichlet directions (♯ND). In the case of ♯ND=4 intersections, the "quark" field q supplied by the defect is a complex scalar field, while q is a fermion field in the case of ♯ND=8 intersections.

Brane	♯ND=4	Embedding
D1	(0| D1 $\perp$ D3)	AdS_2
D3	(1| D3 $\perp$ D3)	$AdS_3\times S^1$
D5	(2| D5 $\perp$ D3)	$AdS_4\times S^2$
D7	(3| D7 $\perp$ D3)	$AdS_5\times S^3$

Brane	♯ND=8	Embedding
D5	(0| D5 $\perp$ D3)	$AdS_2\times S^4$
D7	(1| D7 $\perp$ D3)	$AdS_3\times S^5$

Tab. 1: The possible configurations of 1/2 supersymmetric AdS-branes.

When we focus upon the analysis of the S^5 part, open strings can live on the S^n part ($n = 2,3,4,5$)[10] that is a part of the above-mentioned AdS-branes. In the case of the S^n, we would need the $SO(n+1)_{\rm H}$ coherent states to evaluate the classical sigma model action by taking a continuum limit. We argue that the appropriate choice of the maximal stability subgroup H and the vacuum $|0\rangle$ would be

$$H = SO(n-2)\times SO(2)\,, \qquad |0\rangle = (\underbrace{0,\ldots,0}_{n-1},1,i)\,. \tag{6.1}$$

That is, the coherent states for general AdS-branes are constructed for the coset:

$$SO(n+1)_{\rm H}/[SO(n-2)\times SO(2)]\,. \tag{6.2}$$

By using the coherent state for the coset (6.2),

$$|m\rangle = \exp\left[\sum_{i=1}^{n-1}(a_i M_{i5} + a_{i+4}M_{i6})\right]|0\rangle\,, \tag{6.3}$$

[10] The $n=0$ case has no part in S^5. For the $n=1$ case, the group $SO(2)_{\rm H}$ is abelian and the H becomes trivial. Then the coherent state leads to the BPS vacuum. Hence we will not consider them here.

with $|0\rangle = (0,\ldots,0,1,i)$, we can evaluate the expectation value of the integrable open spin chain Hamiltonian $D_{SO(6)}$. Following the work of Stefanski and Tseytlin [41], let us introduce an antisymmetric imaginary $(n+1)\times(n+1)$ matrix $m^{ij} \equiv \langle m|M^{ij}|m\rangle$. The only difference from the work [41] is the range of the index i . The integrable boundary terms may possibly vanish according to our description and then the expectation value of $H_{l,l+1}$ is evaluated as follows:

$$\langle m|H_{l,l+1}|m\rangle \;=\; \frac{1}{2}\left\{1-\frac{1}{16}(n-1)\right\}\mathrm{Tr}(m_l - m_{l+1})^2 + \frac{1}{32}(n-1)\mathrm{Tr}(m_l^2 - m_{l+1}^2)^2\,. \quad (6.4)$$

After taking a continuum limit, we obtain the following classical sigma model action:

$$\langle m|D_{SO(6)}|m\rangle \to L\int_0^{\pi}\frac{d\sigma}{\pi}\,\frac{\tilde{\lambda}}{32}\left[\mathrm{Tr}(\partial_\sigma m)^2 + \frac{1}{16}(n-1)\mathrm{Tr}(m\partial_\sigma m)^2\right]\,, \quad (6.5)$$

where we have used the formula:

$$\mathrm{Tr}[m^2(\partial_\sigma m)^2] = \frac{1}{2}\mathrm{Tr}(\partial_\sigma m)^2 - \frac{1}{2}\mathrm{Tr}(m\partial_\sigma m)^2\,. \quad (6.6)$$

Here we should notice the identity $\mathrm{Tr}(m\partial_\sigma m)^2 = 0$ and thus the resulting sigma model action is independent of n . That is, the n-dependence appears at the quantum spin chain level but it disappears after taking the continuum limit. As the result, by introducing the complex unit vector V^i, we obtain the sigma model Lagrangian,

$$\mathcal{L} \;=\; -iV^{i*}\partial_t V^i - \frac{1}{2}|D_\sigma V^i|^2\,. \quad (6.7)$$

This expression is identical for each of AdS-branes, except the range of the index i .

The sigma model action (6.7) can be rederived from the string action in the same way as in section 5. Thus it may be argued that the complete agreement of the sigma model action would be valid for general AdS-branes as well as $\mathrm{AdS}_4\times\mathrm{S}^2$. This fact would ensure that the supersymmetric AdS-branes may be included in studies of the correspondence between semiclassical strings and SYM operators.

Moreover, our consideration for coherent states may be generalized for some cases other than AdS-branes, for example, orbifold backgrounds [33,35,36,51]. A study in this direction is favorable and interesting.

7 Conclusion and Discussion

We have discussed the correspondence between long defect operators in the dCFT and open semiclassical strings. The coherent states for the integral open spin chain have been constructed.

In particular, we have given a prescription to treat the boundary terms in the open spin chain Hamiltonian. By evaluating the expectation value of the Hamiltonian with the coherent states and taking the continuum limit, a Landau-Lifshitz (LL) type of sigma model has been obtained. This action has been also derived by starting from the string action with the appropriate gauge fixing and $\kappa \to \infty$. In conclusion, for an open string case, we have found a complete agreement of the sigma model actions in both sides as in closed string cases. It has been also found that an $SO(3)_{\rm H}$ pulsating solution is included as a solution of the derived LL-type sigma model and its energy is completely identical to the result obtained in a different method [45]. In addition, we have discussed the sigma model action in other general AdS-brane cases. When we consider other AdS-branes, the index range of the sigma model variable is different in each of cases according to $\mathrm{S}^n \subset \mathrm{S}^5$ $(n = 2, \ldots, 5)$. However, we have seen that this difference may not affect on the resulting sigma model action and thus we argue that the similar LL-type sigma model would be obtained in other AdS-brane cases.

We have discussed the one-loop dilatation operator for defect operators composed of the $SO(6)$ scalar fields. It is an interesting future work to investigate at two-loop level. The $SO(6)$ sector is closed at one-loop level, but it is not the case at two-loop level. That is, the fields belonging to other sectors may mix into the $SO(6)$ scalar sector. The mixing of fermions occurs at two-loop level, but recently an interesting possibility has been proposed by Minahan [52]. Following the idea of [52], it may be suppressed in taking $L \to \infty$ limit. Thus, the two-loop integrability would possibly hold in this limit. It is important to calculate the two-loop defect interaction in the system we considered and check the consistency to the doubling trick in the string side at two-loop level. We will work in this direction and report the result in the near future [53].

It is also an interesting subject to clarify the connection between the integrability and supersymmetric D-branes (i.e., BPS conditions). The relation between BPS conditions and rotating string solutions is discussed in [54, 55]. A study in this direction would be helpful to investigate such an issue. In addition, it is interesting to clarify the relationship of the integrabilities (Yangian symmetries) of the Landau-Lifshitz type sigma models in closed and open cases (For a closed case, see [56]).

Acknowledgments

We would like to thank H. Fuji, M. Hatsuda, H. Kawamura, D. Tomino and A. Yamaguchi for useful discussion. The work of K. Y. is supported in part by JSPS Research Fellowships for Young Scientists.

References

[1] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113] [arXiv:hep-th/9711200]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428** (1998) 105 [arXiv:hep-th/9802109]; E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2** (1998) 253 [arXiv:hep-th/9802150].

[2] D. Berenstein, J. M. Maldacena and H. Nastase, "Strings in flat space and pp waves from N = 4 super Yang Mills," JHEP **0204** (2002) 013 [arXiv:hep-th/0202021].

[3] R. Penrose, "Any spacetime has a plane wave as a limit," Differential geometry and relativity, Reidel, Dordrecht, 1976, pp. 271-275.
R. Gueven, "Plane wave limits and T-duality," Phys. Lett. B **482** (2000) 255 [arXiv:hep-th/0005061].

[4] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, "A new maximally supersymmetric background of IIB superstring theory," JHEP **0201** (2002) 047 [arXiv:hep-th/0110242].

[5] R. R. Metsaev, "Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background," Nucl. Phys. B **625** (2002) 70 [arXiv:hep-th/0112044]; R. R. Metsaev and A. A. Tseytlin, "Exactly solvable model of superstring in plane wave Ramond-Ramond background," Phys. Rev. D **65** (2002) 126004 [arXiv:hep-th/0202109].

[6] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "A semi-classical limit of the gauge/string correspondence," Nucl. Phys. B **636** (2002) 99 [arXiv:hep-th/0204051].

[7] S. Frolov and A. A. Tseytlin, "Semiclassical quantization of rotating superstring in $AdS_5 \times S^5$," JHEP **0206** (2002) 007 [arXiv:hep-th/0204226].

[8] G. Mandal, N. V. Suryanarayana and S. R. Wadia, "Aspects of semiclassical strings in AdS_5," Phys. Lett. B **543** (2002) 81 [arXiv:hep-th/0206103],
I. Bena, J. Polchinski and R. Roiban, "Hidden symmetries of the $AdS_5 \times S^5$ superstring," Phys. Rev. D **69** (2004) 046002 [arXiv:hep-th/0305116],
L. F. Alday, "Nonlocal charges on $AdS_5 \times S^5$ and pp-waves," JHEP **0312** (2003) 033 [arXiv:hep-th/0310146],
M. Hatsuda and K. Yoshida, "Classical integrability and super Yangian of superstring on $AdS_5 \times S^5$," arXiv:hep-th/0407044.

[9] L. Dolan, C. R. Nappi and E. Witten, "A relation between approaches to integrability in superconformal Yang-Mills theory," JHEP **0310** (2003) 017 [arXiv:hep-th/0308089]; "Yangian symmetry in D = 4 superconformal Yang-Mills theory," arXiv:hep-th/0401243.

[10] G. Arutyunov and M. Staudacher, "Matching higher conserved charges for strings and spins," JHEP **0403** (2004) 004 [arXiv:hep-th/0310182]; "Two-loop commuting charges and the string / gauge duality," arXiv:hep-th/0403077.

[11] S. Frolov and A. A. Tseytlin, "Multi-spin string solutions in $AdS_5 \times S^5$," Nucl. Phys. B **668** (2003) 77 [arXiv:hep-th/0304255]; "Quantizing three-spin string solution in $AdS_5 \times S^5$," JHEP **0307** (2003) 016 [arXiv:hep-th/0306130]; "Rotating string solutions: AdS/CFT duality in non-supersymmetric sectors," Phys. Lett. B **570** (2003) 96 [arXiv:hep-th/0306143].

[12] G. Arutyunov, S. Frolov, J. Russo and A. A. Tseytlin, "Spinning strings in $AdS_5 \times S^5$ and integrable systems," Nucl. Phys. B **671** (2003) 3 [arXiv:hep-th/0307191]; G. Arutyunov, J. Russo and A. A. Tseytlin, "Spinning strings in $AdS_5 \times S^5$: New integrable system relations," Phys. Rev. D **69** (2004) 086009 [arXiv:hep-th/0311004].

[13] A. A. Tseytlin,"Spinning strings and AdS/CFT duality," arXiv:hep-th/0311139.

[14] J. A. Minahan and K. Zarembo, "The Bethe-ansatz for N = 4 super Yang-Mills," JHEP **0303** (2003) 013 [arXiv:hep-th/0212208].

[15] N. Beisert, C. Kristjansen and M. Staudacher, "The dilatation operator of N = 4 super Yang-Mills theory," Nucl. Phys. B **664** (2003) 131 [arXiv:hep-th/0303060].

[16] N. Beisert, S. Frolov, M. Staudacher and A. A. Tseytlin, "Precision spectroscopy of AdS/CFT," JHEP **0310** (2003) 037 [arXiv:hep-th/0308117].

[17] N. Beisert, J. A. Minahan, M. Staudacher and K. Zarembo, "Stringing spins and spinning strings," JHEP **0309** (2003) 010 [arXiv:hep-th/0306139].

[18] V. A. Kazakov, A. Marshakov, J. A. Minahan and K. Zarembo, "Classical / quantum integrability in AdS/CFT," JHEP **0405** (2004) 024 [arXiv:hep-th/0402207].

[19] C. Kristjansen, "Three-spin strings on $AdS_5 \times S^5$ from N = 4 SYM," Phys. Lett. B **586** (2004) 106 [arXiv:hep-th/0402033]; L. Freyhult, "Bethe ansatz and fluctuations in SU(3) Yang-Mills operators," JHEP **0406** (2004) 010 [arXiv:hep-th/0405167]; C. Kristjansen and T. Mansson, "The circular, elliptic three-spin string from the SU(3) spin chain," Phys. Lett. B **596** (2004) 265 [arXiv:hep-th/0406176].

[20] A. V. Belitsky, A. S. Gorsky and G. P. Korchemsky, "Gauge / string duality for QCD conformal operators," Nucl. Phys. B **667** (2003) 3 [arXiv:hep-th/0304028]; A. V. Belitsky, S. E. Derkachov, G. P. Korchemsky and A. N. Manashov, "Quantum integrability in (super) Yang-Mills theory on the light-cone," Phys. Lett. B **594** (2004) 385 [arXiv:hep-th/0403085], "Dilatation operator in (super-)Yang-Mills theories on the light-cone," arXiv:hep-th/0409120.

[21] V. M. Braun, S. E. Derkachov and A. N. Manashov, "Integrability of three-particle evolution equations in QCD," Phys. Rev. Lett. **81** (1998) 2020 [arXiv:hep-ph/9805225]; S. E. Derkachov, G. P. Korchemsky and A. N. Manashov, "Evolution equations for quark gluon distributions in multi-color QCD and open spin chains," Nucl. Phys. B **566** (2000) 203 [arXiv:hep-ph/9909539] and references therein.

[22] N. Beisert and M. Staudacher, "The N = 4 SYM integrable super spin chain," Nucl. Phys. B **670** (2003) 439 [arXiv:hep-th/0307042].

[23] V. A. Kazakov and K. Zarembo, "Classical/quantum integrability in non-compact sector of AdS/CFT," arXiv:hep-th/0410105.

[24] N. Beisert, "The su(2|3) dynamic spin chain," Nucl. Phys. B **682** (2004) 487 [arXiv:hep-th/0310252].

[25] D. Serban and M. Staudacher, "Planar N = 4 gauge theory and the Inozemtsev long range spin chain," JHEP **0406** (2004) 001 [arXiv:hep-th/0401057].

[26] N. Beisert, V. Dippel and M. Staudacher, "A novel long range spin chain and planar N = 4 super Yang-Mills," JHEP **0407** (2004) 075 [arXiv:hep-th/0405001].

[27] G. Arutyunov, S. Frolov and M. Staudacher, "Bethe ansatz for quantum strings," JHEP **0410** (2004) 016 [arXiv:hep-th/0406256];
N. Beisert, "Spin Chain for Quantum Strings," arXiv:hep-th/0409054;
M. Staudacher, "The factorized S-matrix of CFT/AdS," arXiv:hep-th/0412188.

[28] A. Karch and L. Randall, "Localized gravity in string theory," Phys. Rev. Lett. **87** (2001) 061601 [arXiv:hep-th/0105108]; "Open and closed string interpretation of SUSY CFT's on branes with boundaries," JHEP **0106** (2001) 063 [arXiv:hep-th/0105132].

[29] O. DeWolfe, D. Z. Freedman and H. Ooguri, "Holography and defect conformal field theories," Phys. Rev. D **66** (2002) 025009 [arXiv:hep-th/0111135].

[30] J. Erdmenger, Z. Guralnik and I. Kirsch, "Four-dimensional superconformal theories with interacting boundaries or defects," Phys. Rev. D **66** (2002) 025020 [arXiv:hep-th/0203020].

[31] P. Lee and J. Park, "Open strings in PP-wave background from defect conformal field theory," Phys. Rev. D **67** (2003) 026002 [arXiv:hep-th/0203257].

[32] O. DeWolfe and N. Mann, "Integrable open spin chains in defect conformal field theory," JHEP **0404** (2004) 035 [arXiv:hep-th/0401041].

[33] D. Berenstein, E. Gava, J. M. Maldacena, K. S. Narain and H. Nastase, "Open strings on plane waves and their Yang-Mills duals," arXiv:hep-th/0203249.

[34] V. Balasubramanian, M. Huang, T. S. Levi and A. Naqvi, "Open strings from N = 4 super Yang-Mills," JHEP **0208** (2002) 037 [arXiv:hep-th/0204196].

[35] B. Stefanski, Jr., "Open spinning strings," JHEP **0403** (2004) 057 [arXiv:hep-th/0312091].

[36] B. Chen, X. J. Wang and Y. S. Wu, "Integrable open spin chain in super Yang-Mills and the plane-wave / SYM duality," JHEP **0402** (2004) 029 [arXiv:hep-th/0401016]; "Open spin chain and open spinning string," Phys. Lett. B **591** (2004) 170 [arXiv:hep-th/0403004].

[37] N. Beisert, "The complete one-loop dilatation operator of N = 4 super Yang-Mills theory," Nucl. Phys. B **676** (2004) 3 [arXiv:hep-th/0307015].

[38] M. Kruczenski,“Spin chains and string theory,” arXiv:hep-th/0311203.

[39] M. Kruczenski, A. V. Ryzhov and A. A. Tseytlin, “Large spin limit of $AdS_5 \times S^5$ string theory and low energy expansion of ferromagnetic spin chains,” Nucl. Phys. B **692** (2004) 3 [arXiv:hep-th/0403120].

[40] A. Perelomov, “Generalized Coherent States and Their Applications,” Berlin, Germany: Springer (1986) 320 p.; W. M. Zhang, D. H. Feng and R. Gilmore, “Coherent States: Theory And Some Applications,” Rev. Mod. Phys. **62** (1990) 867.

[41] B. Stefanski, Jr., and A. A. Tseytlin, “Large spin limits of AdS/CFT and generalized Landau-Lifshitz equations,” JHEP **0405** (2004) 042 [arXiv:hep-th/0404133].

[42] M. Kruczenski and A. A. Tseytlin, “Semiclassical relativistic strings in S^5 and long coherent operators in $N = 4$ SYM theory,” arXiv:hep-th/0406189.

[43] A. A. Tseytlin, “Semiclassical strings in $AdS_5 \times S^5$ and scalar operators in $N = 4$ SYM theory,” arXiv:hep-th/0407218; “Semiclassical strings and AdS/CFT,” arXiv:hep-th/0409296.

[44] R. Hernandez and E. Lopez, “The SU(3) spin chain sigma model and string theory,” JHEP **0404** (2004) 052 [arXiv:hep-th/0403139].

[45] J. A. Minahan, “Circular semiclassical string solutions on $AdS_5 \times S^5$,” Nucl. Phys. B **648** (2003) 203 [arXiv:hep-th/0209047].

[46] J. Engquist, J. A. Minahan and K. Zarembo, “Yang-Mills duals for semiclassical strings on $AdS_5 \times S^5$,” JHEP **0311** (2003) 063 [arXiv:hep-th/0310188].

[47] H. Dimov and R. C. Rashkov, “Generalized pulsating strings,” JHEP **0405** (2004) 068 [arXiv:hep-th/0404012]; M. Smedback, “Pulsating strings on $AdS_5 \times S^5$,” JHEP **0407** (2004) 004 [arXiv:hep-th/0405102].

[48] A. Mikhailov,“Speeding strings,” JHEP **0312** (2003) 058 [arXiv:hep-th/0311019]; “Slow evolution of nearly-degenerate extremal surfaces,” arXiv:hep-th/0402067; “Supersymmetric null-surfaces,” arXiv:hep-th/0404173; “Notes on fast moving strings,” arXiv:hep-th/0409040.

[49] K. Skenderis and M. Taylor, "Branes in AdS and pp-wave spacetimes," JHEP **0206** (2002) 025 [arXiv:hep-th/0204054].

[50] M. Sakaguchi and K. Yoshida, "D-branes of covariant AdS superstrings," Nucl. Phys. B **684** (2004) 100 [arXiv:hep-th/0310228]; "Notes on D-branes of type IIB string on $AdS_5 \times S^5$," Phys. Lett. B **591** (2004) 318 [arXiv:hep-th/0403243].

[51] K. Ideguchi, "Semiclassical strings on $AdS_5 \times S^5/Z_M$ and operators in orbifold field theories," JHEP **0409** (2004) 008 [arXiv:hep-th/0408014].

[52] J. A. Minahan,"Higher loops beyond the SU(2) sector," arXiv:hep-th/0405243.

[53] Y. Susaki, Y. Takayama and K. Yoshida, to appear.

[54] D. Mateos, T. Mateos and P. K. Townsend, "Supersymmetry of tensionless rotating strings in $AdS_5 \times S^5$, and nearly-BPS operators," JHEP **0312** (2003) 017 [arXiv:hep-th/0309114]; "More on supersymmetric tensionless rotating strings in $AdS_5 \times S^5$," arXiv:hep-th/0401058.

[55] F. Bigazzi, A. L. Cotrone and L. Martucci, "Semiclassical spinning strings and confining gauge theories," Nucl. Phys. B **694**, 3 (2004) [arXiv:hep-th/0403261].

[56] A. Agarwal and S. G. Rajeev, "Yangian symmetries of matrix models and spin chains: The dilatation operator of N = 4 SYM," arXiv:hep-th/0409180.